

VI - error Analysis :

We know that the Earth turns around the sun during 365 days, 05 hours, 48 minutes and 45.5 seconds, therefore the Julian calendar made last the year 11 minutes and 14 seconds (almost) moreover than the solar year, but the pope Gregory XIII has to try to reduce or correct this error, by the 03 days suppression during 400 years.

We note : $e \equiv$ the Julian error during one year.

$c \equiv$ Gregorian correction during one year.

$\delta x = e - c \equiv$ the small error which remains in the course of time
(during one year).

We have :
$$\begin{cases} 1m = \frac{1}{60}h = \frac{1}{60 \times 24}j . \\ 1s = \frac{1}{60}m = \frac{1}{60 \times 60}h = \frac{1}{60 \times 60 \times 24}j . \end{cases} \quad \text{where } j \equiv \text{day} .$$

- For the Julian error, we have then :

$$\begin{aligned} e &= 11m + 14s = 11 \times \frac{1}{60 \times 24}j + 14 \times \frac{1}{60 \times 60 \times 24}j \\ &\cong 0.007638 j + 0.000162 j \cong 0.0078 j . \end{aligned}$$

then :

$$\boxed{e \cong 0.0078 J}$$

- And for the Gregorian correction, we have :

$$\begin{array}{ccc} 3j & \longrightarrow & 400 \text{ years} . \\ c & \longrightarrow & 1 \text{ year} . \end{array}$$

From where : $c = \frac{3}{400}j = 0.0075 j .$

Then :

$$\boxed{c = 0.0075 j}$$

We find then : $\delta x = e - c = 0.0078 j - 0.0075 j \cong 0.0003 j .$

Then :

$$\boxed{\delta x \cong 0.0003 J}$$

- We will seek how much year is necessary they to reach the one day error (01 day).

We have, therefore : $\sum_{n=0}^{n=n_0} \delta x_n = \int_0^{n_0} \delta x_n dn = \left[n \delta x \right]_0^{n_0} = n_0 \cdot \delta x = 1 . / \delta x_i \text{ are equal} .$

From where : $n_0 = \frac{1}{\delta x} \cong \frac{1}{0,0003} \cong 3333.333$ years.

Then :

$$n_0 \cong 3333.333 \text{ years.}$$

Therefore we will have an error of one day during each 3333.333 years ; but this value can be larger, because of uncertainty to the value of δx where its reverse becomes large for a small uncertainty of it.

Then , because of uncertainties , and by reasons of astronomical calculation (homogeneity astronomical), we will take the value of n_0 equal to 4000 years.

Thus, we will approve that 01 day is error which configures during all 4000 years.

In fact, the error which configures during all 4000 years exceeds by a few (01) one day.